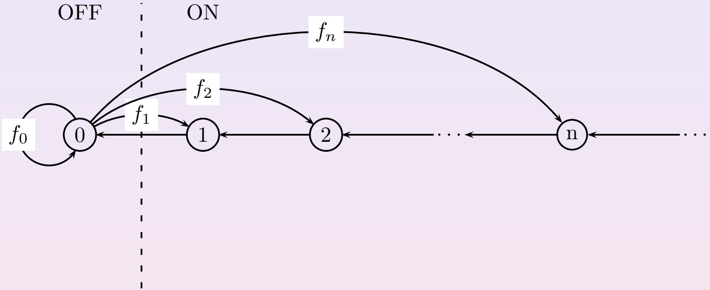
Mathematical models of internet traffic on a link



Richard G. Clegg (richard@richardclegg.org)
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Talk Overview

Motivation

- Several mathematical models exist in the literature which claim to model the internet traffic on a link.
- These models are motivated by the need to model and to predict queuing delay (buffer provisioning).
- In specific many models aim to capture Long-range dependence (LRD).
- How useful are these models in practice?

Problem definition

- Define some stochastic process $\{X_t : t \in \mathbb{N}\}$ where $X_i \in \{0, 1\}$ with 1 representing a packet and 0 an inter-packet gap.
- The stochastic process is parameterised and these parameters can be set to match measurements of "real traffic".
- The simulated traffic should exhibit the same queuing behaviour as the real traffic.

Quick guide to Long-Range Dependence

Long-Range Dependence

LRD is often defined in terms of the autocorrelation function (ACF), $\rho(k)$. A weakly-stationary series has LRD if $\sum_{i=0}^{\infty} \rho(k)$ does not converge. Often it is assumed that

$$\rho(k) \sim Ck^{2(1-H)},$$

where $C > 0$ and $H \in (1/2, 1)$ is known as the Hurst parameter.

- Measured in packets/unit time on internet data [Leland et al '93]. Can cause problems with queuing/delay [Erramilli et al 96].
- This has triggered a huge research effort in LRD based traffic models.

Fractional Gaussian noise (fGn) and fractional Brownian motion (fBm)

Fractional Brownian motion

fBm is a generalisation of Brownian motion (self similar). $B_H(0) = 0$ a.s. $B_H(t)$ is a continuous function of t and

$$\mathbb{P}\left[B_H(t+k) - B_H(t) \leq x\right] = (2\pi)^{-\frac{1}{2}} k^{-H} \int_{-\infty}^{x} \exp\left(\frac{-u^2}{2k^{2H}}\right) du,$$

where $H \in [1/2, 1)$ is the Hurst parameter with $H = 1/2$ being Brownian motion.

Fractional Gaussian noise (fGn) is the process $B_H(t+k) - B_H(t)$ for a given fixed k. Efficient methods exist for generating a fixed length discrete sample $\{B_H(t) : t \in 1, 2, \ldots, N\}$ for a given H and N.

A continous time traffic model from fBm

- fBm can be used as the basis for a contiuous time queuing model [Norros '94].
- Let $\{Y(t) : t \in \mathbb{R}^+\}$ be fBm.
- Let $A(t) = mt + \sqrt{am}Y(t)$ be the arrival process where m is the mean arrival rate and a is a variance parameter.
- Let the queue drain at some constant rate $c > m$.
- Assume an the queue length is not bounded (infinite buffer).
- Results can be stated about the probability of the queue length $\mathbb{P}[Q(t) > x]$ (at least firm lower bounds can be established).

A discrete time traffic model from fGn

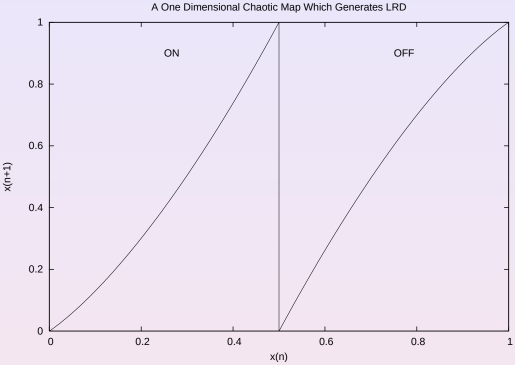
- Let $\{Y(t) : t \in \mathbb{N}\}$ be a discrete time sample from fGn.
- Assume an queue with infinite capacity which drains one unit of traffic per unit time.
- Let $Z(t) = mt + a\sum_{i=1}^{t} Y(u)$ where $m < 1$ is the mean number of arrivals per unit time and a is a variance parameter.
- The arrival process $\{A(t) : t \in \mathbb{Z}^+\}$ is generated by the rules below (initialised with $A(0) = 0$.

$$A(t) = \begin{cases} A(t-1)+1 & Z(t) \geq A(t-1)+1 \\ A(t-1) & \text{otherwise.} \end{cases}$$

- This can be thought of as accumulating work until there is enough to generate a single arrival.
- Open research questions (as far as I am aware):
  1. How can a best be tuned to match real traffic (other than "tweaking")?
  2. How well does this process reflect the nature of fGn?

Iterated Chaotic Map

Iterated double-sided Manneville–Pomeau map.



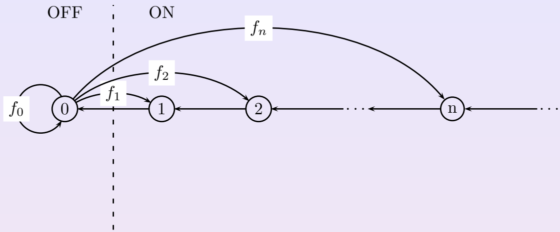
$$x_{n+1} = \begin{cases} x_n + \frac{1-d}{d^{m_1}} x_n^{m_1} & 0 < x_n < d, \\ x_n - \frac{d}{(1-d)^{m_2}} (1 - x_n)^{m_2} & d < x_n < 1, \end{cases}$$

where $x_n, d \in (0,1)$, $m_1, m_2 \in (3/2, 2)$.

Developments from the iterated Chaotic map

- Map produces LRD with $H = max(m_1 - 1, m_2 - 1)$.
- Open research questions (as far as I know):
  1. What is the invariant density of this map? (difficult)
  2. Given m_1 and m_2 what d do I pick to get mean traffic level μ?
- The map becomes easier if we construct a piecewise linear approximation (with an infinite number of pieces).
- The pieces are chosen so that one piece completely maps to the "next one out" (at least as far as the discontinuity — the domain of piece n is (z_n, z_{n+1}) and the range is (z_{n+1}, z_{n+2}) [Wang 1989].
- The symbolic dynamics of the one-sided piecewise linear approximation could be an infinite Markov chain.

The Markov Model

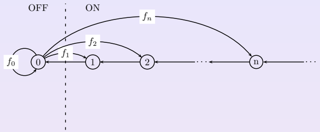


- This is topology of Wang and Clegg/Dodson models.
- If $\{X_t : t \in \mathbb{N}\}$ is generated by chain then generate

$$Y_t = \begin{cases} 0 & X_t = 0 \\ 1 & \text{otherwise.} \end{cases}$$

- Model can be finite or infinite. f_i are transition probs, π_i equilibrium densities.
- Can choose f_i so return times have heavy-tails and get binary series with LRD [Heath et al 1998].

Setting the Transition Probabilities



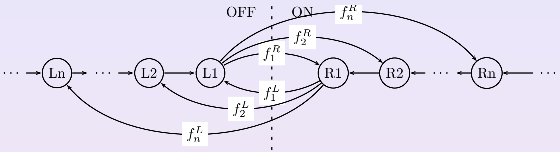
- Two parms α and $\mu = 1 - \pi_0$ if ergodic (conditions easy).
- Find f_k such that $\sum_{i=k}^{\infty} \pi_i \sim Ck^{-\alpha}$.

Transition Probabilities for this Markov model

$$f_k = \begin{cases} \frac{1-\pi_0}{\pi_0}\left[k^{-\alpha} - 2(k+1)^{-\alpha} + (k+2)^{-\alpha}\right] & k > 0 \\ \frac{1-\pi_0}{\pi_0}\left[1 - 2^{-\alpha}\right] & k = 0 \end{cases}$$

- From balance equations $\pi_k = \pi_{k+1} + f_k \pi_0$.
- Thus $\pi_k = \pi_0 \sum_{i=k}^{\infty} f_i$. (Note, if $k = 0$ this says $\pi_0 = \pi_0$).
- For $k > 0$ then $\pi_k = (1 - \pi_0)[k^{-\alpha} - (k+1)^{-\alpha}]$.
- Hence $\sum_{i=k}^{\infty} \pi_i = (1 - \pi_0)k^{-\alpha}$ for $k > 0$ as required.

Arrowsmith/Barenco Model



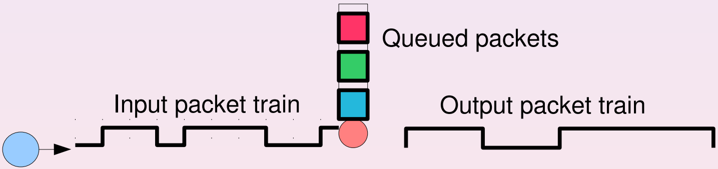
- General class of models described in [Barenco & Arrowsmith '04] proof of strong result giving LRD.
- Think of as double-sided version of Wang topology.
- Could set model to use LRD with Wang or Clegg/Dodson probabilities but theoretical issues cause problem with mean and stability.
- Instead model actual ON/OFF distribution in digitised data.

Models Used

- Simple and tractable packet generation models.
- Models are "clocked" and "binary". Fixed width packets generated at times $n\Delta t : n \in \mathbb{N}$.
- Models used
  1. Poisson process (strictly speaking Bernoulli process) (mean only).
  2. Fractional Gaussian noise model — (mean, "variance" and Hurst parameter).
  3. Wang model [Wang '89], Clegg/Dodson Model [Clegg & Dodson '05] — Markov Modulated process (mean and H).
  4. Model true ON distribution, OFF distribution or both.

Queuing Model

- Assume a single FIFO server with an infinite buffer and output bandwidth b.
- Takes time l/b to process a packet of length l.
- If l is constant then this is a G/D/1 type queue.
- Measure $\mathrm{E}[q]$ the expected queue length (in packets or in bits) as function of b.
- Input to the queue maybe from "real" traffic traces or from models.



Real Traffic Traces

- 100,000 packets from real life traffic sources which give times and packet lengths.
- Establish base case — use arrivals times and lengths as input to queue. Pick b to get approx 10% occupancy.
- Get "digitised" version of real data by only allowing output of fixed l bit packets at times $n\Delta t$.
- CAIDA OC48 data, two sets ($H = 0.6$) from spring 2003. High speed link (2.45 Gb/s). Available from CAIDA website.
- Bellcore data two sets ($H = 0.8$) much beloved historic data from autumn 1989. Available from Internet Traffic Archive.
- QUAINT data ($H = 0.85$) collected at Imperial college April 2003 traffic to and from a heavily loaded router (no results here).

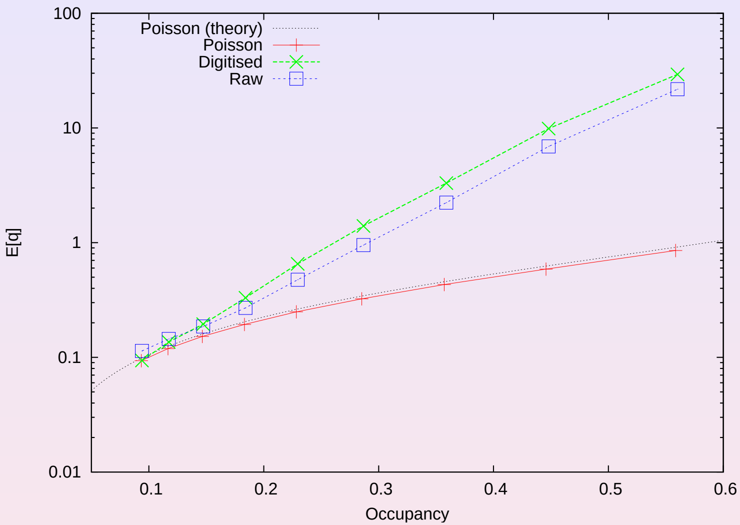
Aside — difficulties of estimating the Hurst parameter

This is the QUAINT data (the least consistent). Code for Hurst estimation available on my website.

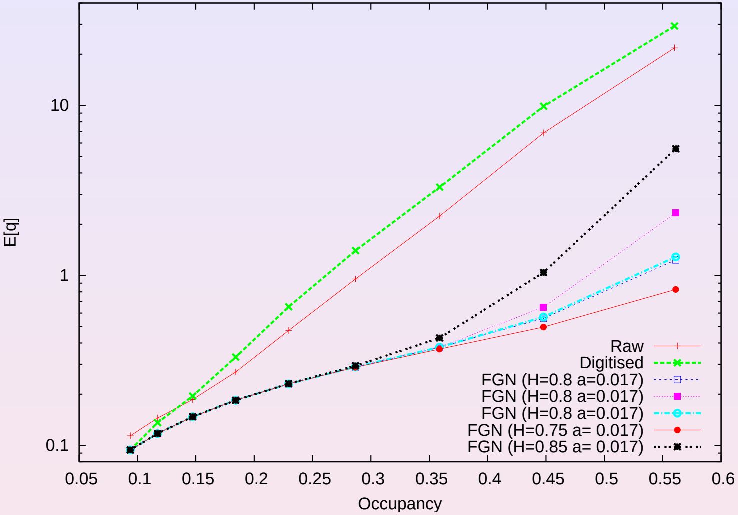
Method	(0.1s bins)	(1s bins)	(10s bins)	(100s bins)
No samples	295969	29597	2960	296
RS plot	0.681	0.752	0.787	0.818
Agg var	0.919	0.860	0.839	0.806
Period.	0.977	0.987	0.870	1.400
Wavelet	0.887	1.105	0.811	0.591
Loc. W.	0.796	0.981	1.019	0.920

- The bin size in theory should not matter (then neither should the choice of estimator).
- Too small and many bins will be empty causing problems for the estimators.
- Too large and there will be insufficient data (rule of thumb, a thousand points is the least I would be comfortable with).

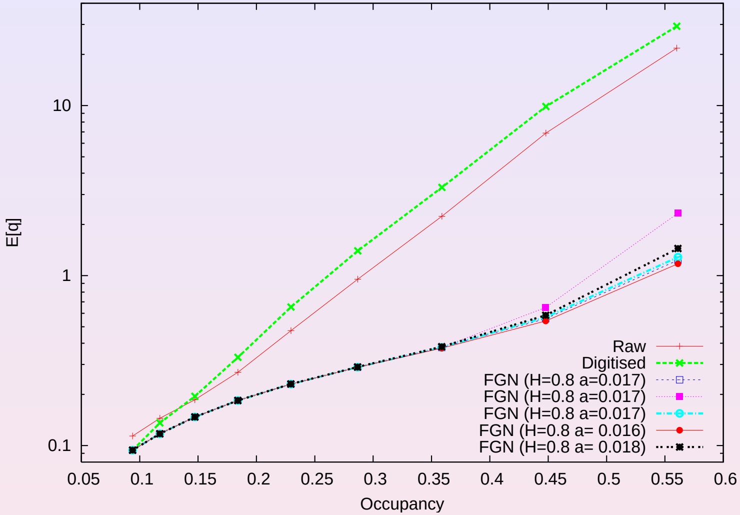
Failure of Poisson modelling (Bellcore data)



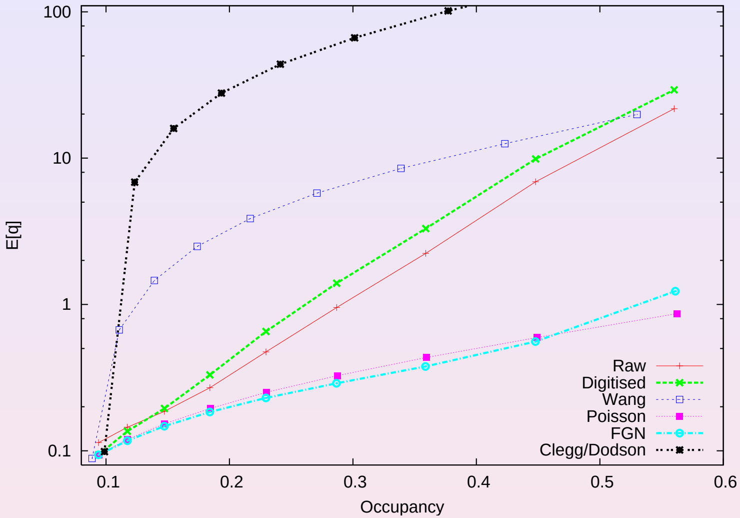
Comparison of FGN modelling (Bellcore data)



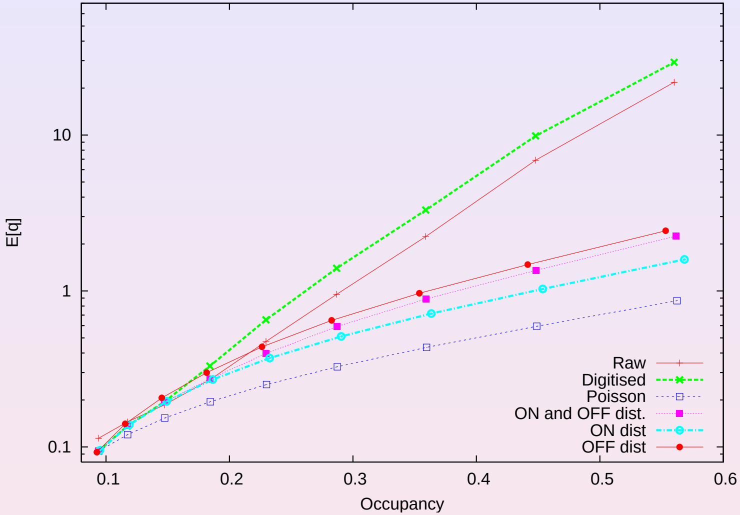
Comparison of FGN modelling 2 (Bellcore data)



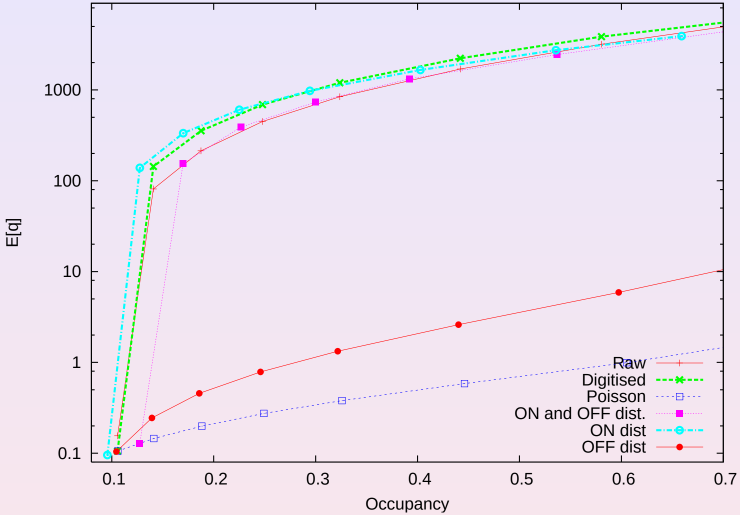
Comparison of all few parameter models (Bellcore data)



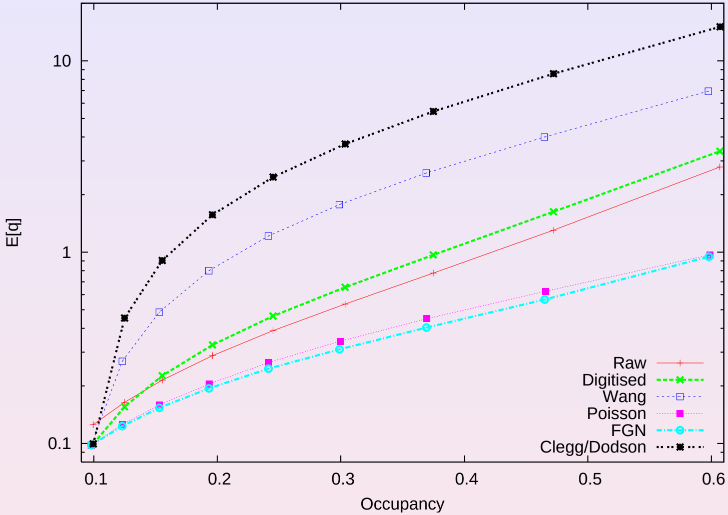
Comparison of multi-parameter models (Bellcore data)



Comparison of multi-parameter models (Bellcore B data)



Comparison of all few parameter models (OC 48 data)



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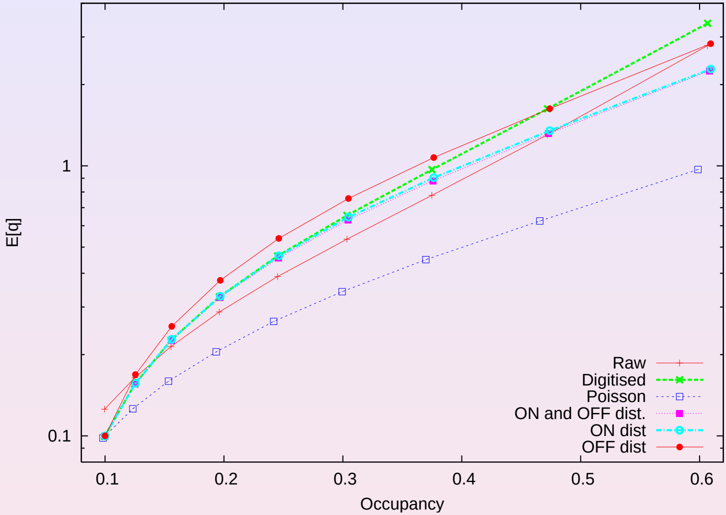
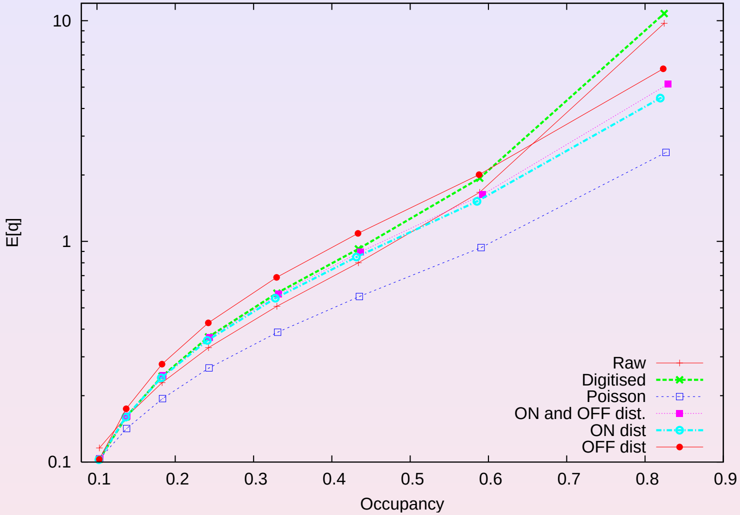


Comparison of multi-parameter models (OC 48 B data)



The mathematical queuing model

- The Markov model can be formulated as a specific queuing model (it belongs to the class of D-BMAP/D/1 models).
- Consider the following discrete time queuing model based on the Markov model.
  1. The system has two states, *on* and *off*.
  2. If in *off* state, prob f_0, next state is *off* state.
  3. If in *off* state, prob f_i next i states *on* then *off*.
  4. If *on* prob. g_n exactly n units of work arrive in iter ($g_0 = 0$).
  5. If *off* no work arrives.
  6. One unit of work consumed per iteration.
- Can we calculate steady state $\mathrm{E}[Q]$ and $\mathbb{P}[Q = q]$?
- This system with the on and off reversed in the chain is the Discrete Batch Renewal process.
- This system with the two sided chain would (I think) be a renewal/reward batch process.

Li's result

Remarkable result from [Li 1993].

Arrival process discrete time MMP with transition matrix $\mathbf{P}$.

Server process deterministic one unit of work per time unit.

Let $\mathbf{Q}(z) = [Q_0(z), Q_1(z), \ldots, Q_n(z)]^T$ be a column vector of the queue length generating function.

$$Q_i(z) = \sum_{q=0}^{\infty} \mathbb{P}\left[Q = q, \text{ state } = i\right] z^q.$$

$\mathbf{Q}(z) = (z-1)[z\mathbf{I} - \mathbf{P}^T\mathbf{G}(z))]^{-1}\mathbf{P}^T\mathbf{B}$, where

$$\mathbf{G}(z) = \mathrm{diag}(A_0(z), A_1(z), \ldots, A_n(z)),$$

$$A_i(z) = \sum_{j=0}^{\infty} z^j \mathbb{P}\left[j \text{ arrivals when in state } i\right],$$

and $\mathbf{B}$ is a boundary column vector (prob. zero arrivals and queue zero in each state).

Implications for my queueing model

Working from Li, I can get a closed form for the equilibrium queue length.

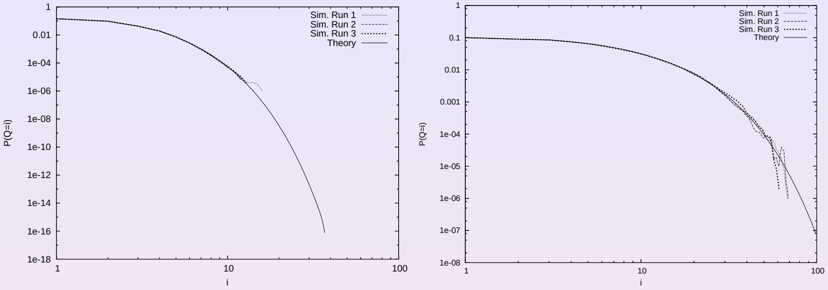
$$\mathrm{E}[Q] = \frac{\overline{g}(\overline{g}-1)\left[\overline{f^2}-\overline{f}^2\right] + \overline{f}(1+\overline{f})\left[\overline{g^2}-\overline{g}^2\right]}{2(1+\overline{f})[1+\overline{f}-\overline{f}\,\overline{g}]},$$

where

$$\overline{f} = \sum_{i=1}^{n} i f_i \text{ and } \overline{f^2} = \sum_{i=1}^{n} i^2 f_i$$
$$\overline{g} = \sum_{i=1}^{m} i g_i \text{ and } \overline{g^2} = \sum_{i=1}^{m} i^2 g_i.$$

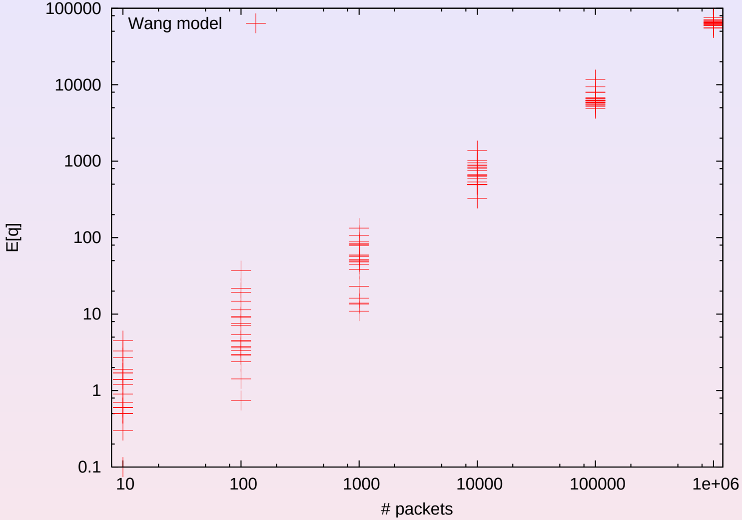
Similarly a series of recursive functions allow calculation of $\mathbb{P}[Q=q]$ in terms of $\mathbb{P}[Q=q-1]$.

Implications for LRD from this model

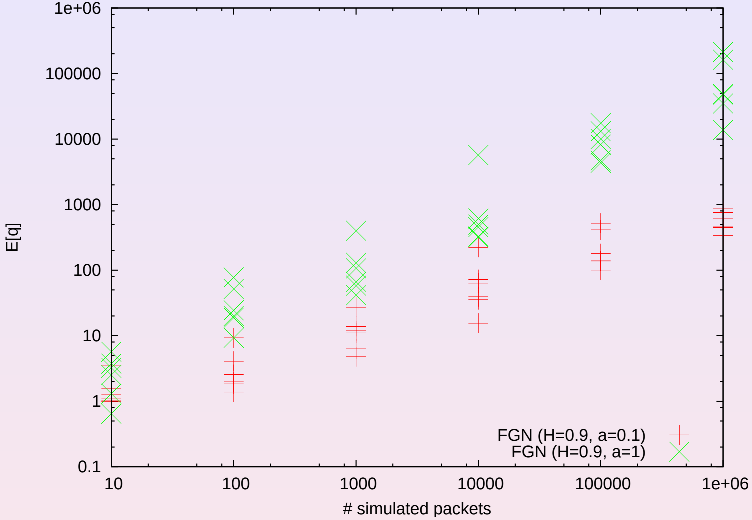


- Matches simulation but numerically unstable as probs shrink.
- But, for (all?) LRD models, $\overline{g^2}$ infinite!
- Model is predicting infinite expected queue length although occupancy below 1.
- Mean queue is infinite even though queue can be empty as large a proportion of the time as we wish. Can this possibly be true? (cf P-K theorem).

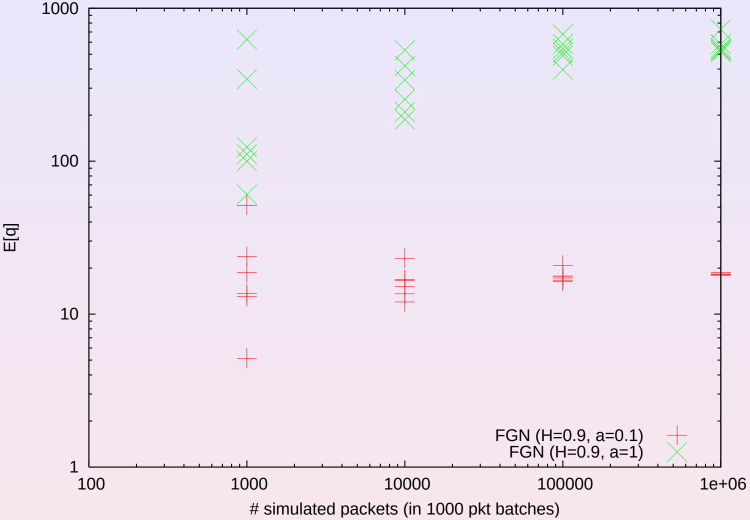
Experimental proof? Wang LRD model.



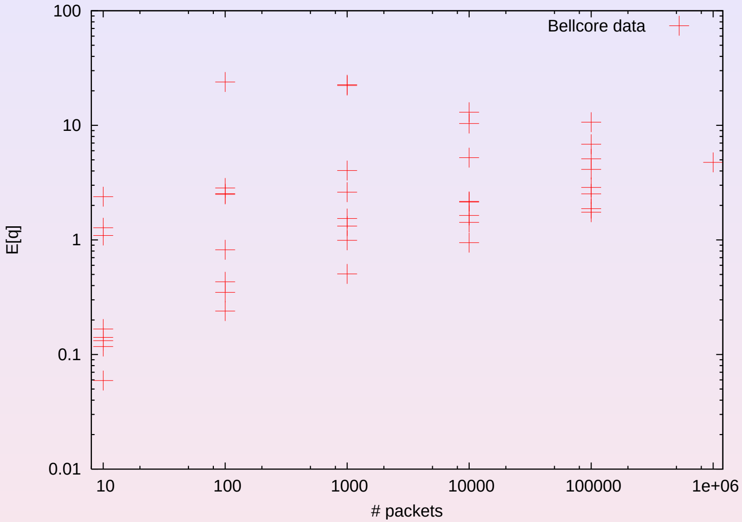
True for other LRD sources? – FGN



Genuine product of LRD? – FGN split into chunks



True for real data?



Conclusions (general conclusions)

- No models with few parameters were close to matching queuing behaviour.
- Getting a simple model to match queuing performance is very difficult.
- The “digitisation” in these models is not the reason for the difference.
- Models which took the distribution of ON burst lengths were sometimes “good enough”.
- The Markov model with ON distribution is fast to run and has easy theoretic answers.
- I need more data and fewer parameters (good models here have many parms).

Conclusions (LRD modelling)

- LRD is a nuisance to work with (poor convergence of mean, hard to measure H) is it fundamental anyway?
- All LRD models matched mean (sort of) and Hurst and for FGN variance but got different wrong answers.
- Real traffic does not queue like LRD traffic.
- The majority of the theoretical results about LRD traffic are $\mathbb{P}[Q > q]$ for an infinite buffer – but this model seems to be predicting infinite delay.
- The very idea of LRD modelling may be fundamentally broken.

Where to now?

- Need data to see how good the on distribution model is in general.
- Can we use theoretical model to match real queue (should be easy?).
- Multi-parameter model is fast to run, easy to implement and has strong theoretical results.
- Simplify multi-parameter models? Other models? (Wavelets? Model ACF?)
- Closed loop models?
  - Pro: Captures importance of TCP feedback mechanism.
  - Anti: Likely to be mathematically intractable. Does complex simulation gain us understanding?
- Open questions:
  - Can we prove that FGN and M-P do not have convergent $\mathrm{E}[Q]$?
  - Is it a feature of all possible LRD models?
  - Is it just a feature of something about my queue model?

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This talk, the author's papers referred to above and the software used are all available online at:

`www.richardclegg.org/`.