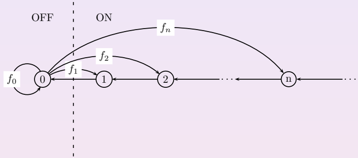
Modelling internet traffic using Markov chains

Scaling laws and their effect on queuing



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(Prepared using LaTeX and beamer.)

Talk Overview

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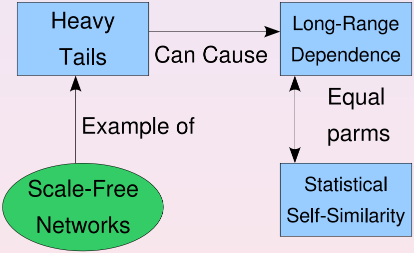
Aims

- Validate simple statistical models of internet traffic against real data.
- Show that models can capture most important statistical parameters of data.
- Show that models can produce traffic with the same queuing performance.

Overview of Power Laws

1. Heavy-Tailed Distribution — Extreme values are more common than expected.
2. Statistical Self-Similarity — Data looks "the same" at all aggregations.
3. Long-Range Dependence — correlations in data last a long time.
4. Scale-Free Networks — Network with some "highly connected" nodes.



- Diagram shows relationships between these power laws.
- There may be other relationships to be discovered.

Long-Range Dependence

Definition of Long-Range Dependence

A weakly-stationary time series is said to be *long-range dependent* (LRD) if the sum $\sum_{k=-\infty}^{\infty} |\rho(k)|$ diverges where $\rho(k)$ is the autocorrelation function. Often a specific form is assumed

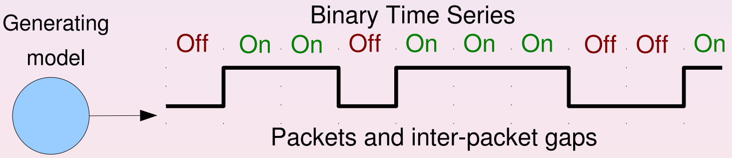
$$\rho(k) \sim Ck^{-\alpha},$$

where $\sim$ means asym. equal as $k \to \infty$, $C > 0$ and $\alpha \in (0, 1)$ are const. Hurst parameter $H = 1 - \alpha/2 \in (1/2, 1)$ is common measure of LRD.

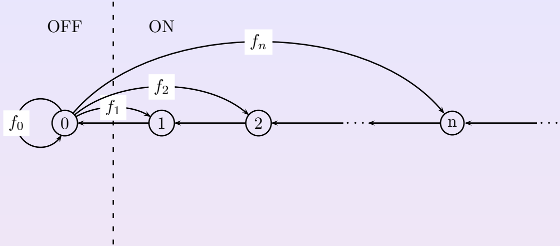
- In 1993 LRD (and self-similarity) was found in bytes/unit time on LAN [Leland et al '93].
- The Hurst parameter is "a dominant characteristic for a number of packet traffic engineering problems" [Erramilli '96].
- Measuring H in real data is a real pain [Clegg '06].

Models Used

- Simple and tractable packet generation models.
- Models are "clocked" and "binary". Fixed width packets generated at times $n\Delta t : n \in \mathbb{N}$.
- Generating Models (listed in chronological order):
  1. Poisson process (strictly speaking Bernoulli process).
  2. Fractional Brownian Motion model.
  3. Wang model [Wang '89] — Markov Modulated process.
  4. Pseudo Self-Similar Traffic (PSST) [Robert et al '97] — MMP.
  5. Arrowsmith/Barenco [Barenco & Arrowsmith '04] — MMP (no results given).
  6. Clegg/Dodson [Clegg & Dodson '05] — MMP.



The Markov Model



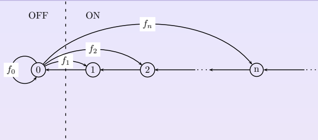
- This is topology of Wang and Clegg/Dodson models.
- If $\{X_t : t \in \mathbb{N}\}$ is generated by chain then generate

$$Y_t = \begin{cases} 0 & X_t = 0 \\ 1 & \text{otherwise.} \end{cases}$$

- The f_i are trans. prob. and the π_i equilibrium densities.
- Want simple values of f_i to work with.
- Choose f_i so return times have heavy-tails and get binary series with LRD [Heath et al 1998].

Clegg/Dodson Model

- Two parms α and $\mu = 1 - \pi_0$ if ergodic (simple conds known).
- Find f_k such that $\sum_{i=k}^{\infty} \pi_i \sim Ck^{-\alpha}$.

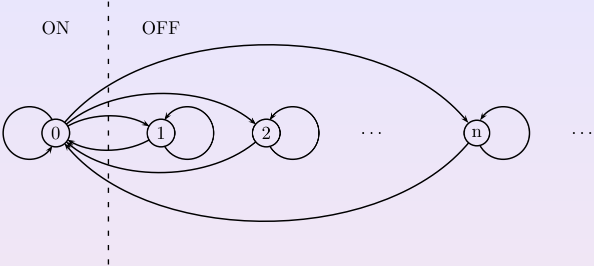


Transition Probabilities for this Markov model

$$f_k = \begin{cases} \frac{1-\pi_0}{\pi_0}\left[k^{-\alpha} - 2(k+1)^{-\alpha} + (k+2)^{-\alpha}\right] & k > 0 \\ \frac{1-\pi_0}{\pi_0}\left[1 - 2^{-\alpha}\right] & k = 0 \end{cases}$$

- From balance equations $\pi_k = \pi_{k+1} + f_k \pi_0$.
- Thus $\pi_k = \pi_0 \sum_{i=k}^{\infty} f_i$. (Note, if $k = 0$ this says $\pi_0 = \pi_0$).
- For $k > 0$ then $\pi_k = (1 - \pi_0)[k^{-\alpha} - (k+1)^{-\alpha}]$.
- Hence $\sum_{i=k}^{\infty} \pi_i = (1 - \pi_0)k^{-\alpha}$ for $k > 0$ as required.
- Formal proof of LRD exists (H related to α).
- Wang model similar but f_i different.

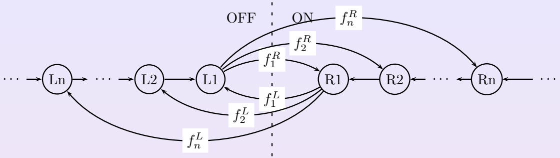
Pseudo-Self-Similar Traffic Model (PSST)



- Introduced in [Robert et al '97] no proof of LRD.
- Parameters: q relates to mean a has no obvious interpretation.

$$\mathbf{P} = \begin{bmatrix} \Sigma_0 & \frac{1}{a} & \frac{1}{a^2} & \cdots \\ \frac{q}{a} & \Sigma_1 & 0 & \cdots \\ \left(\frac{q}{a}\right)^2 & 0 & \Sigma_2 & \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

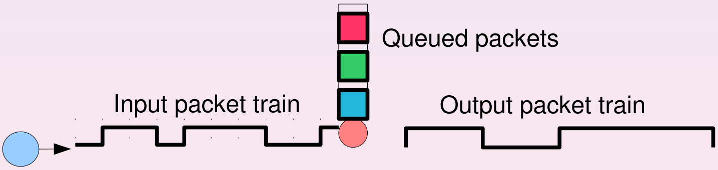
Arrowsmith/Barenco Model



- General class of models described in [Barenco & Arrowsmith '04] proof of strong result giving LRD.
- Think of as double-sided version of Wang topology.
- Can set heavyness of tail for ON and OFF periods.
- Could use Wang or Clegg/Dodson probabilities but theoretical issues cause problem with mean of traffic and stability (no results here).
- This should not be taken as criticism of this family of models.

Queuing Model

- Assume a single FIFO server with an infinite buffer and output bandwidth b.
- Takes time l/b to process a packet of length l.
- If l is constant then this is a G/D/1 type queue.
- Measure $\mathrm{E}[q]$ the expected queue length (in packets or in bits) as function of b.
- Input to the queue maybe from "real" traffic traces or from models.



Real Traffic Traces

- 100,000 packets from two real life traffic sources which give times and packet lengths.
- Establish base case — use arrivals times and lengths as input to queue. Pick b to get approx 10% occupancy.
- Get "digitised" version of real data by only allowing output of fixed l bit packets at times $n\Delta t$.
- All models are two parameter (except Bernouilli) — try to match base μ (and hence var) and H.

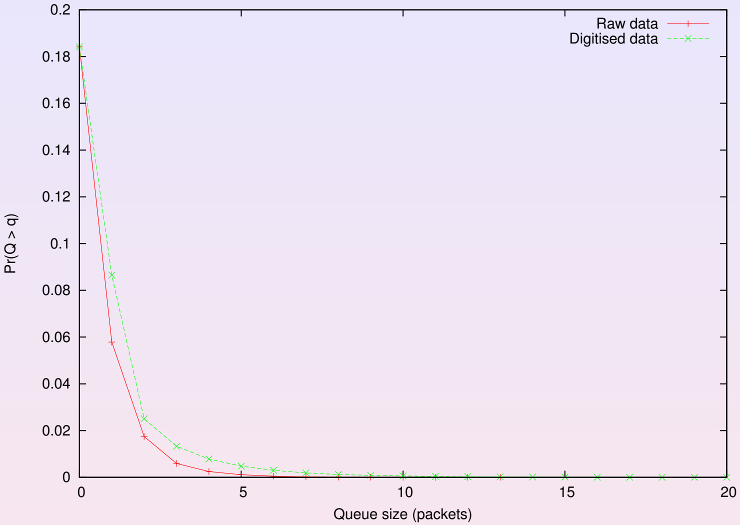
CAIDA OC48 data ($H = 0.6$)

- Data from April 2003.
- High speed link (2.45 Gb/s).
- Available from: `www.caida.org/data/passive`.

Bellcore data ($H = 0.8$)

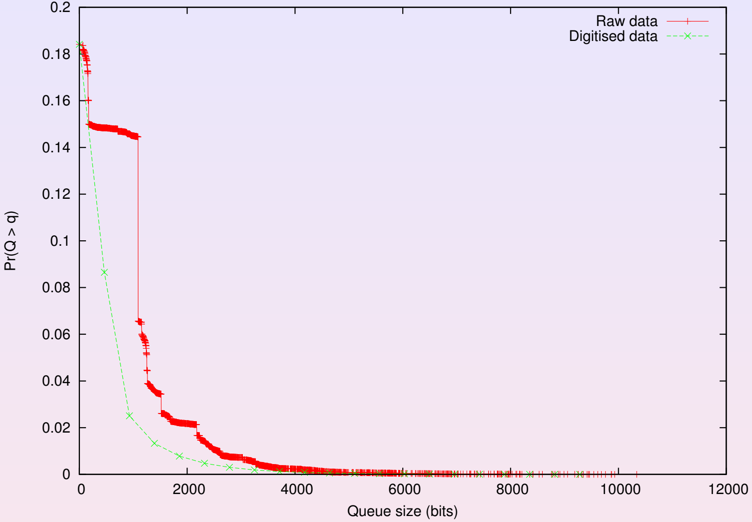
- Much beloved historic data (Aug 1989).
- Available from: `ita.ee.lbl.gov/html/contrib/BC.html`

Bellcore digitisation (by packet)

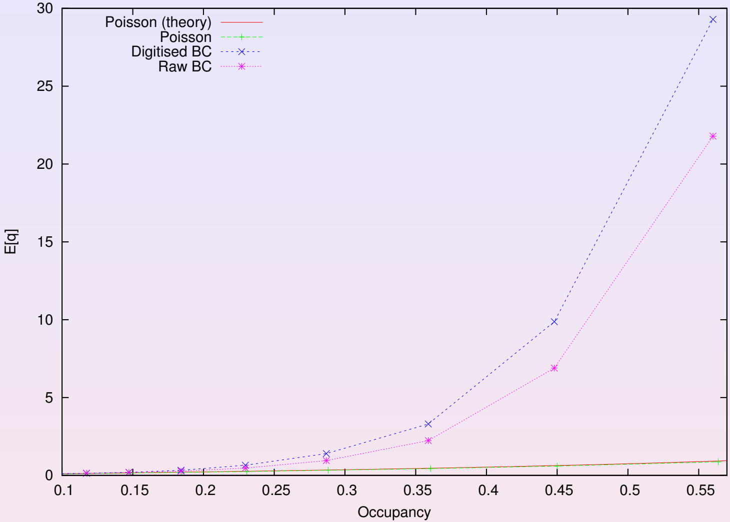


1 - PDF of queue size in packets before and after digitisation of the Bellcore data (queued at half bw).

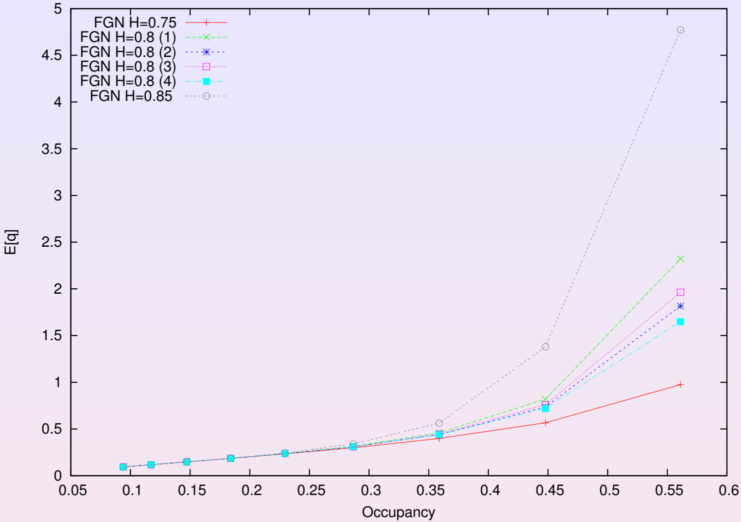
Bellcore digitisation (by bits)



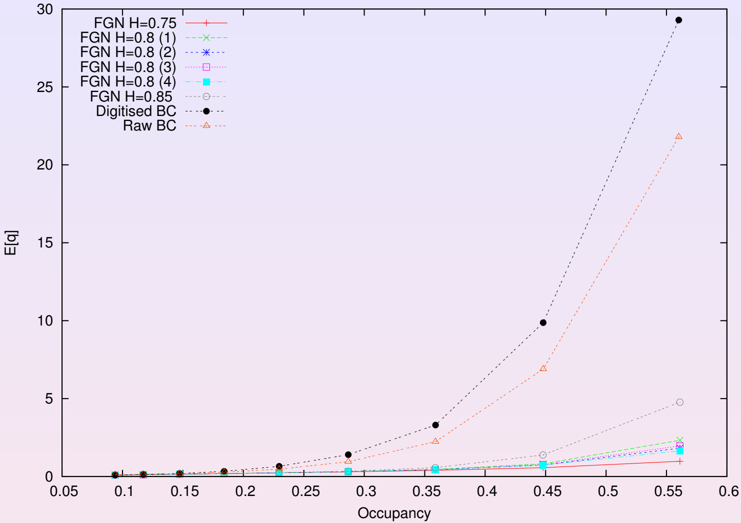
1 - PDF of queue size in bits before and after digitisation of the Bellcore data (queued at half bw).



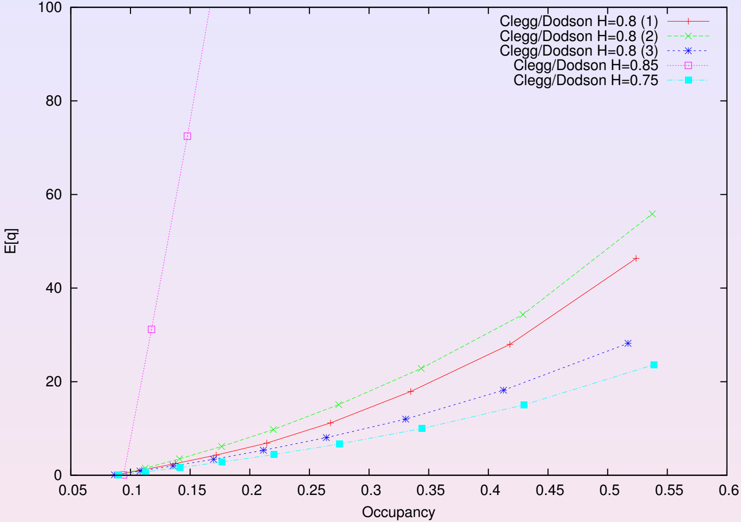
Poisson versus real data (theory line is from P-K theorem).



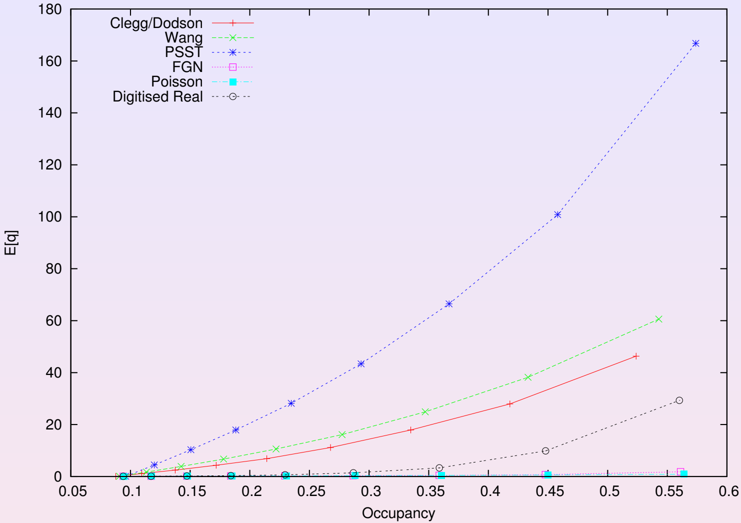
FGN (several realisations with $H = 0.8$ and one each of $H = 0.75$ and $H = 0.85$).



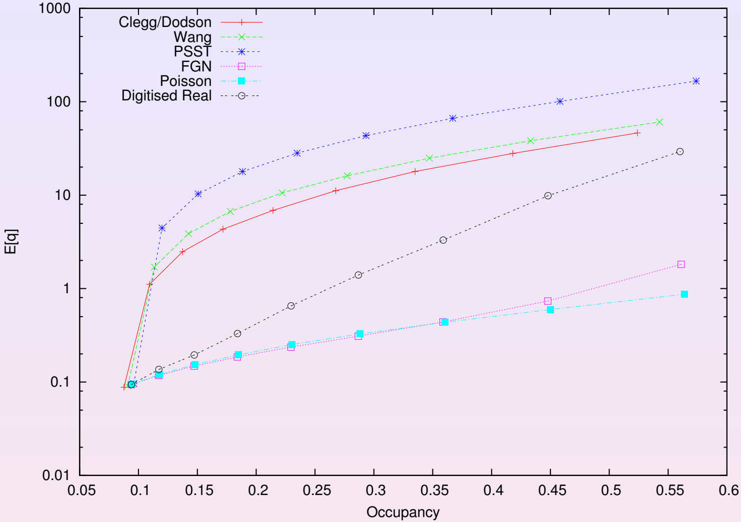
FGN compared with real data.



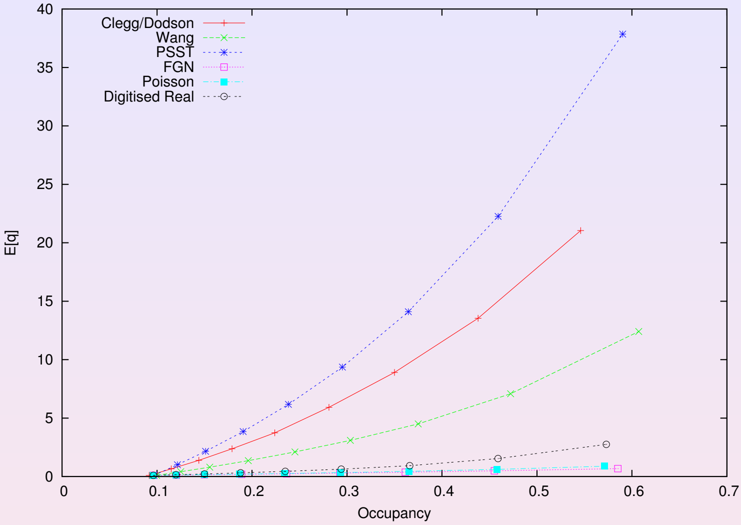
Clegg/Dodson model. Three realisations with $H = 0.8$ one with $H = 0.85$ and one with $H = 0.75$.



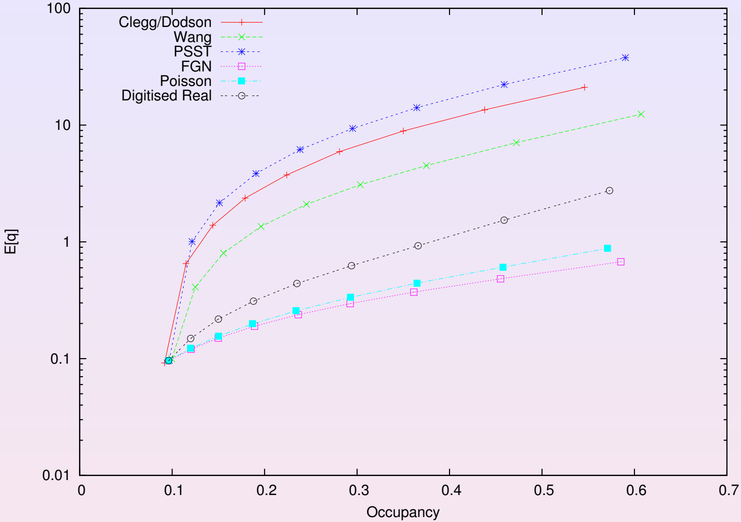
All models compared with digitised data.



All models compared with digitised data (y logscale).



CAIDA data. All models compared with digitised data.



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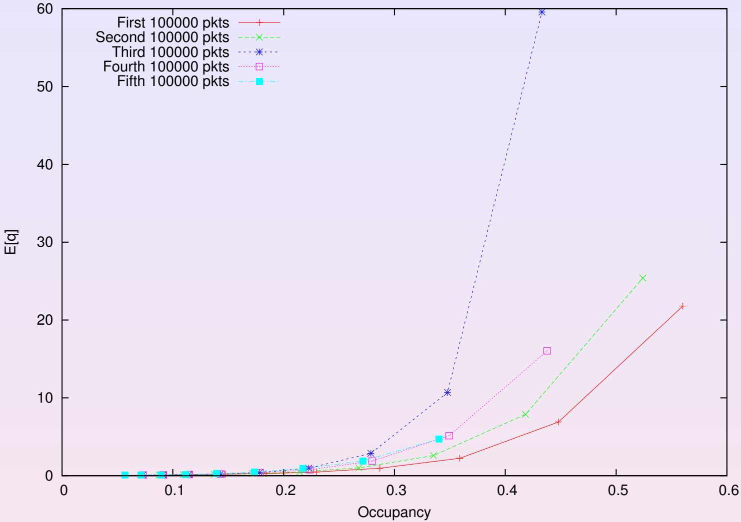

The next four blocks of 100,000 packets on Bellcore (raw data queuing performance).

Conclusions

- LRD is a nuisance to work with (poor convergence of mean, hard to measure H) is it fundamental anyway?
- Theoretical studies may have been looking at the "wrong" occupancy levels.
- All models matched mean (sort of) and Hurst once aggregated (except PSST).
- The PSST model is very peculiar — I needed to use the reverse of it anyway. (Non-Hurst LRD?)
- No models were even close to matching queuing behaviour.
- Getting a simple model to match queuing performance is very difficult.
- Real traffic is variable in ways which simple models cannot be.
- Hurst parameter can be "naughty" or "nice" [Neidthardt '98].

Where to now?

- Multi-parameter models? (Multi-fractal wavelet model? Variants of Arrowsmith/Barenco model?)
  - Pro: Captures more parameters of traffic.
  - Pro: Mathematics is interesting.
  - Anti: Mathematics is much more difficult (accuracy versus understanding).
- Closed loop models?
  - Pro: Captures importance of TCP feedback mechanism.
  - Anti: Likely to be mathematically intractable.
  - Anti: Does complex simulation gain us understanding?
- What am I missing? (User behaviour? Network behaviour? Misunderstanding theory?)
- Definitely more research required.

References

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This talk, the author's papers referred to above and the software used are all available online at:

`www.richardclegg.org/`.