Networks II – Worksheet One

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Basic Networking

Assume for simplicity throughout this section that 1KB = 1000 bytes (octets) – note that in fact, 1KB = 1024 bytes. Similarly assume 1MB = 1000KB, 1GB = 1000MB and so on. (Note that Kb refers to kilobits and KB refers to kilobytes).

Question 1. Yann and Richard want to copy 6GB of files from their homes to their office. Richard elects to use a modem and transfers the files at 512Kb/sec (assume that the modem operates at this data rate constantly and only data is sent — ignore the effects of packet headers and so on). Yann elects to copy the files to a writable CD (capacity 600MB). It takes him 15 minutes to travel home to his office, 15 minutes to write a CD (copying from files on disk) and 5 minutes to read a CD (copying files on the CD to the disk). If both start off in their office, how long do each take to copy all the data? What is the effective bandwidth in KB/sec for each?

Question 2. List two differences between the OSI reference model and the TCP/IP model. List two ways in which they are the same.

Question 3. Which layer of the OSI model handles:

1. Breaking data into packets.
2. Determining which route to use through a subnet.

Question 4 (*). Consider the following hosts:

- `wobbegong.spurious.ac.uk 128.100.59.16`
- `greatwhite.spurious.ac.uk 128.100.59.17`
- `cookiecutter.spurious.ac.uk 128.100.63.25`
- `mako.spurious.ac.uk 128.100.62.25`
- `tiger.spurious.ac.uk 128.100.1.52`

Which hosts are on the same subnet given the following netmasks:

1. `255.255.255.224`
2. `255.255.255.0`
3. `255.255.254.0`

4. 255.255.192.0
5. 255.255.193.0

Why is the last netmask illegal?

Question 5 (*). A 10MB message is transferred using TCP/IP. The maximum packet size is 1KB including all the headers. How many packets must be sent? What fraction of the bandwidth was wasted on headers?

Basic Queuing Theory and Poisson Processes

Question 6. Two communication nodes 1 and 2 send files to another node 3. Files from 1 and 2 require, on average, R_1 and R_2 time units for transmission respectively. Node 3 processes a file from node $i(i = 1, 2)$ in an average of P_i time units and then requests another file from either node 1 or 2 according to some rule (which is left unspecified). If λ_i is the throughput of node i in files sent per unit time then what is the region of all feasible throughput pairs (λ_1, λ_2) [That is, what values of λ_1 and λ_2 mean that this system can serve files without the queue growing forever.]?

Question 7. Consider K independent sources of packets where the interarrival times of each source are exponentially distributed (that is each source is a Poisson process) with the kth source having mean λ_k. If these packet streams are merged (assuming no delay in doing so), prove that the K independent sources form a Poisson process with mean $\lambda = \lambda_1 + \lambda_2 + \cdots + \lambda_K$.

Question 8. A packet source either emits or does not emit a single packet every microsecond with the probability of emiting a packet being p. Let $(X_j)_{j\geq 0}$ be a timeseries (of zeros and ones) representing this packet stream. A counter records the number of packets seen every N microseconds — $S_N = X_1 + X_2 + \cdots + X_N$. If N has a Poisson distribution with mean λ show that the number of packets counted (S_N) has a Poisson distribution with mean λp.

Question 9 (*). The input to a router is a Poisson Process with parameter λ. The router sends packets down one of its K output streams. Each packet arriving at the router is assigned without delay to a stream chosen at random with a probability p_i that the packet is assigned to stream i (naturally, $\sum_{i=1}^{K} p_i = 1$). Prove that the ith stream is Poisson with parameter $\lambda_i = p_i\lambda$.

Question 10 (*). Packets arrive at a router at a rate of 25 per second. Packets take 5 milliseconds to process. 50 percent of packets are considered urgent and immediately forwarded without queuing. The remaining 50 percent are queued for an average of 20 milliseconds before forwarding. What is the average number of packets in the system (both being processed and being queued)?

Basic Markov Chains

Question 11. Suppose that $(X_n)_{n\geq 0}$ is Markov $(\lambda, \mathbf{P})$. If $Y_n = X_{kn}$ where $(k \in \mathbb{N})$ then show that $(Y_n)_{n\geq 0}$ is Markov $(\lambda, \mathbf{P}^k)$.

Question 12. Show that a point inside or on an equilateral triangle of unit height can be used to represent a distribution in a three state Markov chain in the sense that the sum of the distances of a given point in the triangle to the three sides must always equal unity.

Question 13. A flea hops about the vertices of a triangle. It is twice as likely to hop clockwise as anti-clockwise. Write down $\mathbf{P}$ for the Markov chain. What is the probability that it returns to its starting point after n hops?

Question 14 (*). An octopus is trained to choose object A from a pair of objects A and B by repeated trials. The octopus maybe in one of three states of mind:

1. It is untrained and picks A or B at random
2. It is trained and always picks A but may forget its training.
3. It is trained and always picks A and will never forget its training.

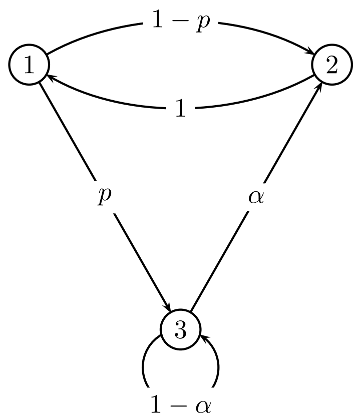
After each trial it is rewarded for success and may change its state accordingly. The transition matrix between states is given by:

$$\mathbf{P} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{12} & \frac{5}{12} \\ 0 & 0 & 1 \end{bmatrix}$$

Assuming that the octopus is in state 1 before trial 1. What is the probability that it is in state 1 before trial $n+1$? What is the probability that it correctly picks item A on trail $n+1$?

The suggestion is made that a two state Markov chain with constant transition probabilities is sufficient to describe the octopus. Discuss briefly this possibility with reference to your calculated value for the probability of picking A on the $n+1$th trial.

Question 15 (*). Consider the Markov chain shown below.



1. Write down $\mathbf{P}$.
2. Under what conditions (if any) will the chain be irreducible, aperiodic, ergodic?
3. Find the equilibrium probability vector $\boldsymbol{\pi}$.
4. What is the mean recurrence time for state 2.
5. Find values of α and p such that $\pi_1 = \pi_2 = \pi_3$.